

# The enigma of dark matter and its possible connection with Wigner's infinite spin representations

dedicated to the memory of Robert Schrader

Bert Schroer

present address: CBPF, Rua Dr. Xavier Sigaud 150,  
22290-180 Rio de Janeiro, Brazil

permanent address: Institut für Theoretische Physik  
FU-Berlin, Arnimallee 14, 14195 Berlin, Germany

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## Abstract

Positive energy ray representations of the Poincaré group are naturally subdivided into three classes according to their mass and spin content:  $m>0$ ,  $m=0$  finite helicity and  $m=0$  infinite helicity. For a long time the localization properties of the massless infinite spin class remained unknown before it became clear that such matter does not permit compact spacetime localization and its generating covariant fields are localized on semi-infinite spacelike strings.

It is shown that such matter cannot interact with normal matter within the conceptual setting of QFT since any such coupling leads to a total delocalization in higher perturbative orders. However, as any positive energy matter, it interacts with gravity. Its inert behavior with respect to ordinary matter makes it an ideal candidate for dark matter.

## 1 Wigner's infinite spin representation and string-localization

Wigner's famous 1939 theory of unitary representations of the Poincaré group  $\mathcal{P}$  was the first systematic and successful attempt to classify relativistic particles according to the *intrinsic* principles of relativistic quantum theory [1]. As we know nowadays, his massive and massless spin/helicity class of positive energy ray representations of  $\mathcal{P}$  does not only cover all known particles, but their "covariantization" [2] leads also to a complete description of all covariant point-local free fields. For each representation there exists a field of lowest short distance dimension; all other "elementary" fields are obtained by applying derivatives, and by forming Wick-ordered products of elementary fields one arrives at the "local equivalence class" of composite fields.

The only presently known way to describe interactions in four-dimensional Minkowski space is to start from a scalar interaction density in terms of Wick-products of free fields with the lowest short distance dimension and use it as the starting point of the cutoff- and regularization-free causal perturbation theory [3]. These free fields do not have to be Euler-Lagrange fields; perturbative QFT can be fully accounted for in terms of interaction densities defined in terms of free fields obtained from Wigner's representation theory without referring to any classical parallelism.

All positive energy representations are "induced" from irreducible representations of the "little group". This subgroup of the Lorentz group is the stability group of a conveniently chosen reference momentum on the forward mass shell  $H_+$ , respectively the forward surface of the light cone  $V_+$ . For  $m > 0$  this is a rotation subgroup of the Lorentz group and for  $m = 0$  the noncompact Euclidean subgroup  $E(2)$ . Whereas the massive representation class ( $m > 0, s = \frac{n}{2}$ ), of particles with mass  $m$  and spin  $s$  covers all known massive particles (the first Wigner class), the massless representations split into two quite different classes.

For the finite helicity representations the  $E(2)$  subgroup of Lorentz-"translations" are trivially represented ("degenerate" representations), so that only the abelian rotation subgroup  $U(1) \subset E(2)$  remains; this accounts for the semi-integer helicity  $\pm |h|$ ,  $|h| = \frac{n}{2}$  (the second Wigner class). The third Wigner class consists of *faithful* unitary representations of  $E(2)$ . Being a noncompact group, they are necessarily *infinite dimensional* and their irreducible components are characterized in terms of a continuous Pauli-Lubanski invariant  $\kappa$ .

Since the Pauli-Lubanski invariant for massive representations is related to the spin as  $\kappa^2 = m^2 s(s+1)$  one may at first think that the properties of this infinite spin matter can be studied by considering it as a limit  $m \rightarrow 0, s \rightarrow \infty$  with  $\kappa$  fixed. However it turns out that  $(m, s)$  spinorial fields do not possess such an infinite spin limit. Our main result concerning the *impossibility of quantum field theoretical interactions between WS with normal matter* depends among other things on the impossibility of such an approximation; for this reason we will refer to these representations briefly as the "Wigner stuff" (WS). This terminology is also intended to highlight some of the mystery which surrounded this class for the more than 6 decades after its discovery and which also the present paper does not fully remove.

For a long time the WS representation class did not reveal its quantum field theoretic localization properties. The standard group theoretical covariantization method to construct intertwiners [2], which convert Wigner's unitary representations into point-like covariant wave functions and their associated quantum fields, did not work for the WS representations. Hence it is not surprising that attempts in [4] (and more recently in [5]), which aim at the construction of wave equations and Lagrangians, fell short of solving the issue of localization. In fact an important theorem [6] dating back to the 70s showed that it is not possible to associate pointlike Wightman fields with these representations.

Using the concept of modular localization, Brunetti, Guido and Longo showed that WS representations permit to construct subspaces which are "modular localized" in arbitrary narrow spacelike cones [7] whose core is a semi-infinite string. In subsequent work [8] [9] such generating string-local covariant fields were explicitly constructed in terms of modular localization concepts. In the same paper attempts were undertaken to show that such string-local fields cannot have point-local composites. These considerations were strengthened in [10]. A rigorous proof which excludes the possibility of finding compact localized subalgebras (generating point-local fields) was finally presented in a seminal paper by Longo, Morinelli and Rehren [11].

Being a positive energy representation, WS shares its stability property and its ability to couple to gravity with the other two positive energy classes; hence it cannot be dismissed from the outset as being unphysical. In view of the unsolved problems concerning the relation of the increasing amount of unexplained astrophysical data concerning the still mysterious dark matter, it is tempting to explore the possibility of a connection between this (according to some physicist) "biggest enigma of the 21st century" and the theoretical puzzle of the WS class. This re-opens a problem which Weinberg temporarily closed in the first volume of his textbook by stating that "nature does not use the WS representations" [2].

The aim of the present paper is to convert the question of whether nature uses WS matter into a theoretical problem of understanding its largely unexplored properties. Such a theoretical question is meaningful because QFT (in contrast to quantum mechanics) is fundamental in the sense that the wealth of its physical consequences can be traced back to different manifestations of its underlying *causal localization principle*.

In the context of quantum theory this principle is much more powerful than its classical counterpart. The concept of *modular localization* permits to address structural problems of QFT in a completely intrinsic way which avoids the use of "field-coordinatizations". An illustration of the power of this relatively new concept is the proof of existence of a certain class of two-dimensional models starting from the observations that certain algebraic structures in integrable  $d=1+1$  models can be used to construct modular localized wedge algebras [12]. In the work of Lechner and others this led to existence proofs for integrable models with nontrivial short distance behavior together with a wealth of new concepts (see the recent review [13] and literature cited therein). Even in renormalized perturbation theory modular localization has become useful in attempts to replace local gauge theory in Krein space by string-local fields in Hilbert space [14].

In [7] it was essential to extract localization properties directly in the form of modular localized subspaces since Weinberg's group theoretic method of constructing covariant local field within the standard intertwiner formalism does not work for WS.

In an unpublished previous note [15] I tried to address the problem of a possible connection between WS and dark matter. But the recent gain of knowledge from modular localization with respect to attempts to unite WS with normal matter under the conceptual roof of AQFT in [11], as well as new insights coming from perturbative studies of couplings involving string-local fields [14] [16], led to a revision of previous ideas.

In [11] it was shown that the attempt to unite normal matter together with WS in a *nontrivial* way<sup>1</sup> under the conceptual roof of AQFT leads to an unexpected (suspicious looking) loss of the so-called "Reeh-Schlieder property" for compact localized observable algebra. The R-S property states that the set of state vectors obtained by the application of operators from a compact localized subalgebra of local observables to the vacuum is "total" in the vacuum Hilbert space. The possibility to manipulate large distance properties of states in the vacuum sector by applying operators localized in a compact spacetime region  $\mathcal{O}$  is considered to be a universal manifestation of vacuum polarization.

It plays an important role in the DHR superselection theory [17] and its breakdown in the presence of WS asks for further clarification. It turns out that, different from point-local interactions where the power-counting requirement  $d_{sd}^{int} \leq 4$  for renormalizability is the only requirement for the perturbative existence of a model, string-local interactions must fulfill an additional quite restrictive condition which prevents their *total delocalization* in higher orders.

The main result of the present paper is that this additional condition cannot be fulfilled in couplings of WS to normal matter. The reason is that in contrast to massless finite helicity matter which can be obtained as a massless limit from string-local massive matter, the class of WS remains completely isolated; in particular it is not simply the  $m \rightarrow 0$ ,  $s \rightarrow \infty$  limit of string-local covariant spin  $s$  fields with fixed Pauli-Lubanski invariant  $\kappa^2 = m^2 s(s+1)$ .

This leaves only the possibility that, apart from interactions with gravity as a consequence of the positive energy property, WS cannot interact with normal matter. In mathematical terminology: normal matter tensor-factorizes with WS and the Reeh-Schlieder property is that of a compact localized observable subalgebra in the tensor factor of normal matter.

A world in which the WS matter only reacts with gravity may be hard to accept from a philosophical viewpoint. But after we have gotten used to chargeless leptons which only couple to the rest of the world via weak and gravitational interactions, the step to envisage a form of only gravitationally interacting kind of matter is not as weird as it looks at first sight; in particular when astrophysical observations reveal that on the one hand the presence of a new form of matter is indispensable for attaining a gravitational balance in agreement with galactic observations, but at the same time require a high degree of reactive inertia ("dark") with respect to ordinary matter.

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<sup>1</sup>Excluding the trivial possibility of a tensor product of WS with the world of ordinary matter.

Apart from the fact that any positive energy matter couples to gravity, Wigner's  $(m, s)$  classification contains no information about possible interactions (strong, electromagnetic, weak). As will be argued in the following, the noncompact localization properties permit however to support the idea that interactions of WS with normal matter are not possible; ignoring gravitation, WS is inert. In contrast to the local observables of noncompact string-like massless tensor potentials interacting among themselves or with  $s < 1$  matter, covariant string-local fields contain no local observables and hence the presence of string-local states created by WS fields acting on the vacuum cannot be registered in particle counters. The possible galactic presence of WS is restricted to pure gravitational manifestations.

The paper is organized as follows.

The next section presents a "crash course" on Wigner's theory of positive energy representations of the Poincaré group including the explicit construction of string-local WS free fields and their two-point functions.

The third section highlights an important restriction on renormalizable couplings involving string-local fields.

In section 4 it is explained why this perturbative restriction can not be fulfilled for WS.

The concluding remarks point at problems arising from the identification of WS with dark matter.

## 2 Matter as we (think we) know it and Wigner's infinite spin "stuff"

The possible physical manifestations of WS matter can only be understood in comparison to normal matter. Hence before addressing its peculiarities it is necessary to recall the localization properties of free massive and finite helicity zero mass fields.

It is well known that all point-local massive free fields can be described in terms of matrix-valued functions  $u(p)$  which intertwine between the creation/annihilation operators of Wigner particles [2]. Their associated covariant fields are of the form

$$\psi^{A,\dot{B}}(x) = \frac{1}{(2\pi)^{3/2}} \int (e^{ipx} u^{A,\dot{B}}(p) \cdot a^*(p) + e^{-ipx} v^{A,\dot{B}}(p) \cdot b(p)) \frac{d^3p}{2p_0} \quad (1)$$

The intertwiners  $u(p)$  and their charge-conjugate counterpart  $v(p)$  are rectangular  $(2A+1)(2B+1) \otimes (2s+1)$  matrices which intertwine between the unitary  $(2s+1)$ -component Wigner representation and the covariant  $(2A+1)(2B+1)$  dimensional spinorial representation labeled by the semi-integer  $A, \dot{B}$  which characterize the finite dimensional representations of the covering of the Lorentz group  $SL(2, c)$ . the  $a^\#(p), b^\#(p)$  refer to the Wigner particle and antiparticle creation/annihilation operators and the dot denotes the scalar product in the  $2s+1$  dimensional spin space.

For a given physical spin  $s$  there are infinitely many spinorial representation-indices of the homogeneous Lorentz group; their range is restricted by

$$\left| A - \dot{B} \right| \leq s \leq A + \dot{B}, \quad m > 0 \quad (2)$$

For explanatory simplicity we restrict our subsequent presentation to integer spin  $s$ ; for half-integer spin there are similar results.

All fields associated with integer spin  $s$  representation can be written in terms of derivatives acting on symmetric tensor fields ( $A = B$ ) of degree  $s$  with lowest short distance dimension  $d_{sd}^s = s + 1$ . For  $s = 1$  one obtains the divergence-less Proca vector potential  $A_\mu^P$  with  $d_{sd} = 2$ , whereas for  $s = 2$  the result is a divergence-free traceless symmetric tensor  $g_{\mu\nu}$  with  $d_{sd} = 3$  etc.

Free fields can also be characterized in terms of their two-point functions whose Fourier transformation are tensors in momenta instead of intertwiners. For  $s = 1$  one obtains

$$\langle A^P(x) A^P(x') \rangle = \frac{1}{(2\pi)^3} \int e^{-ip(x-x')} M_{\mu\nu}^P(p) \frac{d^3p}{2p^0}, \quad M_{\mu\nu}^P(p) = -g_{\mu\nu} + \frac{p_\mu p_\nu}{m^2} \quad (3)$$

and for higher spin the  $M$ 's are symmetric tensors formed from products of  $g_{\mu\nu}$  and products of  $p$ 's ( $P$  stands interchangeably for "Proca" or "point-like")

For  $m = 0$  and finite integer helicity  $h$  the two dimensional  $\pm|h|$  helicity representation replaces the  $2s + 1$  component spin. Despite this difference, the covariant fields turn out to be of the same form (1), except that (2) is now replaced by the more restrictive relation

$$|A - \dot{B}| = |h|, \quad m = 0 \quad (4)$$

which excludes all the previous tensor potential but permits field strengths which are tensors of degree  $2|h|$  and  $d_{sd} = |h| + 1$  with mixed symmetry properties. This is well-known in case of  $|h| = 1$  where there exist no massless point-local vector potential  $A = 1/2 = \dot{B}$  associated to the electromagnetic field strength. The absence of point-local tensor potentials in (4) results from a clash between point-local spin  $s$  tensor potentials and the Hilbert space positivity. Gauge theory substitutes the non-existent point-local Hilbert space vector potential by one in an indefinite Krein space and the prescriptions by which one extracts a physical subtheory lead to gauge theory.

The problem can be resolved in two ways; either one sacrifices positivity or one let the Hilbert space positivity determine the tightest localization consistent with positivity, which turns out to be localization on semi-infinite space-like strings  $x + \mathbb{R}_+ e$ ,  $e^2 = -1^2$ . The first case only leads to a physically restricted theory for which all gauge dependent fields are physically void. The advantage is only computational since massless fields are easier to cope with (however the explicit extraction of the physical data with the help of the BRST ghost formalism remains somewhat involved). The perturbation theory of string-local fields turns out to be more demanding, but as a reward one obtains a full QFT in which (as in  $s < 1$  interactions) all fields are physical (though only few represent local observables).

String-local tensor potentials also exist for massive fields. The string-local counterpart of the point-local massive two-point functions (3) turn out to be

$$M_{\mu\nu}^s(p; e, e') = -g_{\mu\nu} - \frac{p_\mu p_\nu e \cdot e'}{(p \cdot e - i\varepsilon)(p \cdot e' + i\varepsilon)} + \frac{p_\mu e_\nu}{p \cdot e - i\varepsilon} + \frac{p_\nu e'_\mu}{p \cdot e' + i\varepsilon} \quad (5)$$

Its massless limit is of the same form, except that the momentum  $p$  is on the boundary of the positive lightlike surface  $H_0^+$  of the forward light cone  $H_m^+$ ; This has to be taken into account in the Fourier transformation to  $x$ -space. The more complicated form as compared to the  $-g_{\mu\nu}$  in the Feynman gauge setting is the prize to pay for upholding positivity. The best description of the interacting massless theory is to first calculate the renormalized massive correlation functions and then take their massless limit. This has the advantage of performing perturbation theory in the simple Wigner Fock particle space and leaving the reconstruction of the massless limit (in which this physical description of the Hilbert space is lost) to the application of Wightman's reconstruction theorem to the limiting correlation functions.

Massive theories are simpler from a conceptual viewpoint because the presence of a mass gap permit to use the tools of scattering theory and the identification of the Hilbert space with a Wigner Fock space.

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<sup>2</sup>Beware that there is no relation between string-local fields and string theory. Whereas the change from point-local to string-local fields for  $s \geq 1$  is required in order to uphold Hilbert space positivity, ST has no conceptual compass, it is the result of a playful spirit to extend the game of QFT and has no relation to its modular localization.

Whereas it is plausible that the asymptotic short distance behavior of the Hilbert space setting is correctly accounted for in terms of the asymptotic freedom properties of gauge theories, the problems related to long-distance properties as confinement remain outside the physical range of the gauge setting.

There is a very efficient way to derive the relation between the point-local Proca potential and its string-local counterpart. Integrating the latter along the space-like direction  $e$ , one obtains a string-local scalar field  $\phi(x, e)$

$$\begin{aligned}\phi(x, e) &:= \int_0^\infty d\lambda e^\mu A_\mu^P(x + \lambda e) = \frac{1}{(2\pi)^{3/2}} \int (e^{ipx} u(p, e) \cdot a(p) + h.c.) \frac{d^3 p}{2p_0} \\ u(p, e) &:= u(p) \cdot e \frac{1}{ip \cdot e}, \quad M^{\phi, \phi} = \frac{1}{m^2} - \frac{e \cdot e'}{(p \cdot e - i\varepsilon)(p \cdot e + i\varepsilon)}\end{aligned}\tag{6}$$

where the inner product in the first line refers to the 3-dim. spin space and the denominator is simply the Fourier transform of the Heavyside function. The  $\phi$  two-point function can either be computed from carrying out the line integrals on the Proca two-point function or by using the intertwiner. The string-local vector is defined as

$$A_\mu(x, e) := \int_0^\infty e^\nu F_{\mu\nu}(x + \lambda e) ds, \quad F_{\mu\nu} := \partial_\mu A_\nu^P - \partial_\nu A_\mu^P\tag{7}$$

which leads to the two-point function (5).

The three fields turn out to be linearly related

$$A_\mu(x, e) = A_\mu^p(x) + \partial_\mu \phi(x, e)\tag{8}$$

which either can be derived from the previous definition or by defining the three fields in terms of their intertwiners in which case one obtains the relation as a linear relation between intertwiners. The equation looks like a gauge transformation; indeed the extension of this relation to interactions with matter fields suggests a relation between point-local  $\psi(x)$  and its string-local counterpart which has the expected exponential form  $\psi(x, e) = \psi(x) \exp \phi$ . But in contrast to gauge theory these relations intertwine between string-local fields and their more singular point-local siblings in the same string-local relative localization class; in fact this formula, after making it precise in terms of normal products may be seen as the definition of an  $e$ -independent  $d_{sd} = \infty$  counterpart of a polynomially bounded string-local field.

This construction can be extended to all integer spin fields. The divergence-free Proca potential is replaced by divergence- and trace-free symmetric tensor potentials  $A_{\mu_1, \dots, \mu_s}$  of tensor rank  $s$ . Iterated integration along a space-like direction  $e$  leads to  $s$  string-local  $\phi$  tensor fields of lower rank

$$\phi_{\mu_1 \dots \mu_k}(x, e) = \int d\lambda_1 \dots d\lambda_{s-k} e^{\nu_1} \dots e^{\nu_{s-k}} A_{\nu_1, \dots, \nu_{s-k}, \mu_1, \dots, \mu_k}(x + \lambda_1 e + \dots \lambda_{s-k} e)\tag{9}$$

Again one can construct the  $e$ -dependent intertwiners from these relations. The extension of (8) spin  $s$  relates the point-local tensor-potential  $A^P$  to its string-local counterpart and  $s$  string-local tensor escorts  $\phi_{\dots}(x, e)$  of  $A$ .

$$A_{\mu_1, \dots, \mu_s}(x, e) = A_{\mu_1, \dots, \mu_s}^P(x) + \sum_{k=1}^s \partial_{\mu_1, \dots} \partial_{\mu_k} \phi_{\mu_k+1, \dots, \mu_s}\tag{10}$$

The appearance of these lower spin  $\phi$ -escorts is characteristic for the change of massive point-local fields into their string-local siblings acting in a Hilbert space. They are important new fields which depend on the same degrees of freedom (the Wigner creation/annihilation operators) as the other two operators. For  $s = 1$  the scalar field  $\phi$  may be seen as the QFT analog of the bosonic Cooper pairs which are the result

of a reorganization of the condensed matter degrees of freedom in superconductivity<sup>3</sup> which appear at low temperature. Without the formation of Cooper pairs from existing condensed matter degrees of freedom it is not possible to convert the long-range classical vector potentials into its short range counterparts within the superconductor (as anticipated by London). The relation between long range massless and short range massive potentials requires the presence of the  $\phi$  in fact it is not possible to formulate massive QED in Hilbert space (i.e. in terms of string-local potentials) without the presence of these escorts (no need of additional  $H$  fields for massive vector potentials).

The reason why additional degrees of freedom in the form of  $H$ -fields are indispensable in the case of self-interacting massive vectormesons is quite deep but bears no relation to physical spontaneous symmetry breaking, for details we refer to even though the formal prescriptions in terms of Mexican hat potentials lead to the same result as the induction of  $H$ -self-interactions by requiring the  $e$ -independence of the  $S$ -matrix. The presence of  $H$  fields is necessary in order to preserve the second order power-counting bound through a compensation mechanism [14] [16].

The individual lower spin escort fields in (10) have no massless limit, but together with the Proca tensor potentials their presence is necessary for the construction of the massive string-local potential; all these fields are relatively local and act in the same Wigner-Fock Hilbert space. Only the correlation functions of the degree  $s$  string-local tensor potential possess a massless limit.

The  $A$  in (10) is related to the point-local field strength

$$F_{\mu_1 \dots \mu_s, \nu_1 \dots \nu_s} = a s \{ \partial_{\mu_1} \dots \partial_{\mu_s} A_{\nu_1, \dots, \nu_s} \}_{\mu, \nu} \quad (11)$$

where the antisymmetrization imposes antisymmetry between the  $\mu - \nu$  pairs. As in the previous case of the vector potential (7), the string-local tensor potentials with fulfill (10) can be obtained in terms of  $s$  iterated integrations along  $e$  starting from the field strength. The field strength tensor is the lowest rank tensor field<sup>4</sup>. With appropriate changes these results have analogs for semi-integral spin.

Before passing to the string-local fields of the WS class it may be helpful to collect those properties which turn out to be important for a comparison with finite spin string-local fields.

- Whereas pointlike massive tensor potentials have short distance dimension  $d_{sd} = s + 1$ , their string-local counterparts have  $d_{sd} = 1$  independent of spin. Hence there are always first order string-local interaction densities within the power-counting limit  $d_{sd}^{int} \leq 4$
- String-local tensor potentials are smooth  $m \rightarrow 0$  limits of their massive counterpart. They inherit the  $d_{sd} = 1$  from their massive counterpart. The lowest point-local fields in the same representation class are  $d_{sd} = 2s$  field strengths (tensors of rank  $2s$  with mixed symmetry properties).
- The point-local  $d_{sd}^K = 1$  zero mass vector-potentials  $A_\mu^K$  of local gauge theory act in an indefinite metric Krein space. The physical prize for resolving the clash between point-like localization and Hilbert space positivity is the scarification of the Hilbert space. The physical range of  $s = 1$  gauge theory is restricted to local observables which act in a smaller Hilbert space which does not include fields. *The latter contain no information about the physical relation between Hilbert space positivity and Einstein-causal physical localization.* The absence of local observables excludes the use of gauge theory for WS.

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<sup>3</sup>This remark is intended to counteract the claim that one needs additional  $H$ -fields in order to "fatten vector potentials by swallowing Goldstone particles". The physical reason why  $H$ 's are needed in the presence of self-interacting massive vector mesons is quite different (see below).

<sup>4</sup>For  $s = 2$  this tensor has the same mixed symmetry property as the linearized Riemann tensor whereas the symmetric second rank tensor deserves to be denoted as  $g_{\mu\nu}$ .

Whereas the two-point functions of point-local massive free fields are polynomial in  $p$ , their string-local counterparts have a rational  $p$ -dependence (5) (6). The family of all WS interwiners for a given Pauli-Lubanski invariant  $\kappa$  has been computed in [8] their two point-functions are *transcendental functions* of  $p, e$  which are boundary values of from  $Im(e) \in V^+$ .

A particular simple WS intertwiner with optimal small and large momentum space behavior (corresponding to the minimal choice for string-local massless  $s \geq 1$ ) has been given in terms of an exponential function in [10]

$$u(p, e)(k) = \exp i \frac{\vec{k}(\vec{e} - \frac{p_-}{e_-} \vec{p}) - \kappa}{p \cdot e} \quad (12)$$

here  $k$  is a two-component vector of length  $\kappa$ ; the Hilbert space on which Wigner's little group  $E(2)$  acts consists of square integrable functions  $L^2(k, d\mu(k) = \delta(k^2 - \kappa^2)dk)$  on a circle of radius  $\kappa$ . The vectors arrow on  $e$  and  $p$  refer to the projection into the 1-2 plane and the  $e_-, p_-$  refer to the difference between the third and zeroth component. The most general solution of the intertwiner relation differs from this special one by a function  $F(p \cdot e)$  which is the boundary value of a function which is analytic in the upper half-plane [8].

The two-point function is clearly a  $J_0$  Bessel function. The calculation in [10] was done in a special system. Writing its argument in a covariant form one obtains <sup>5</sup>

$$M^{WS}(p, e) \sim J_0(\kappa |w(p, e)|) \exp - i\kappa \left( \frac{1}{p \cdot e - i\varepsilon} - \frac{1}{p \cdot e' + i\varepsilon} \right) \quad (13)$$

$$\text{with } w^2(p, e) = -\left( \frac{e}{e \cdot p - i\varepsilon} - \frac{e'}{e' \cdot p + i\varepsilon} \right)^2$$

The exponential factor compensates the singularity of  $J_0$  at  $e \cdot p = 0$ . Note that the Pauli-Lubanski invariant  $\kappa$  has the dimension of a mass so that the argument of the two-point function of the string-local field has the correct engineering dimension  $d_{eng} = 1$  of a quantum field.

The main purpose of this calculation is to convince the reader that there exist explicitly known transcendental WS intertwiner and two-point functions whose associated propagators with a well behaved ultraviolet and infrared behavior. As already mentioned, the physical reason why these fields are excluded from appearing in interaction densities is that the higher orders lead to a complete de-localization in higher; the necessary condition which they violate will be explained in the next section.

### 3 The problem of maintaining higher order string-localization

It is well known that the only restriction for point-local interaction densities is the power-counting inequality  $d_{sd}^{int} \leq 4$ . Since the minimal short distance dimension of point-local spin  $s$  fields<sup>6</sup> is  $s + 1$ , there are no point-local renormalizable interactions involving  $s \geq 1$  fields. String-local free fields on the other hand have an  $s$ -independent short distance behavior with  $d_{sd} = 1$  so that one always can find polynomials of maximal degree 4 which represent interaction densities within the power-counting limitation.

However such formally renormalizable string-local interaction densities come with a physical hitch. Unless they fulfill an additional requirement, the string-localization cannot be maintained in higher orders. In that case the result will be a *complete delocalization*. Before explaining why an interaction between WS and ordinary matter is irreconcilable with the causal localization principles of QFT, it is helpful to understand the condition for the preservation of string-localization in models with finite spin.

<sup>5</sup>I am indepted to Henning Rehren for showing me the covariantization of Köhler's result.

<sup>6</sup>Fields (without the added specification "gauge") are always acting in Hilbert space.



As a simple nontrivial illustration we start with a string-local interaction density  $L$  massive QED

$$L = A_\mu(x, e)j^\mu(x) \quad (14)$$

Here  $A_\mu(x, e)$  is a massive string-local vector potential (5) and  $j^\mu$  is the conserved current of a massive complex scalar field. These fields act in a Wigner-Fock Hilbert space of the corresponding Wigner particles and since  $d_{sd}(A_\mu) = 1$ ,  $d_{sd}(j^\mu) = 4$  the short distance dimension of  $L$ ,  $d_{sd}^L(L) = 4$  stays within the power-counting bound  $d_{sd}^{int}$  of renormalizability<sup>7</sup>.

If we now iterate this interaction according to Bogoliubov's time-ordered operator formalism for the calculation of either the on-shell S-matrix or the off-shell correlation functions, we find that the number of  $e$ 's in inner propagators (every  $e$  in  $A_\mu(x, e)$  fluctuates separately) increases and the integration of the inner  $x$ 's over all spacetime leads to a spread of the strings  $x + \mathbb{R}_+e$  over all spacetime. The desired situation of a directional independent S-matrix and off-shell correlations which only depend on the  $e_i$  of the correlated fields (and not on  $e_i$ 's of inner propagators) cannot be maintained.

The way to prevent total delocalization and secure the  $e$ -independence of the S-matrix is as follows. One starts from the point-local interaction with  $d_{sd}^{int} = 5$  and uses (8) to rewrite it as

$$\begin{aligned} L^P &= A^P \cdot j = L - \partial^\mu V_\mu, \quad V_\mu(x, e) := \partial_\mu \phi j^\mu \\ \int L^P d^4x &= \int L d^4x, \quad i.e. \quad S^{(1)} = S_P^{(1)} = S_S^{(1)} \end{aligned} \quad (15)$$

where the second line follows since in the presence of a mass gap the divergence of  $V_\mu$  does not contribute to the adiabatic limit which represents the first order S-matrix. The gain is two-fold. On the one hand we have expressed the S-matrix in terms of the renormalizable string-local interaction  $L$  by disposing the most singular  $d_{sd}(\partial \cdot V) = 5$  contribution at infinity (boundary terms do not contribute in massive models). This implies that the first order S-matrix is  $e$ -independent.

For the following it is convenient to formulate the  $e$ -independence in terms of a differential form calculus on the  $d = 1 + 2$  dimensional directional de Sitter space. The differential form of the relation (8) reads

$$\begin{aligned} d_e A_\mu &= \partial_\mu u, \quad u = d_e \phi \\ d_e(L - \partial^\mu V_\mu) &= d_e L - \partial^\mu Q_\mu, \quad Q_\mu := d_e V_\mu \end{aligned} \quad (16)$$

In order to secure the directional independence in higher orders we must implement the  $e$ -independence in the following form (second order, for simplicity of notation):

$$d(TLL' - \partial^\mu TV_\mu L' - \partial^\mu TLV'_\mu + \partial^\mu \partial^\nu TV_\mu V'_\nu) = 0, \quad i.e. \quad S^{(2)} \text{ is } e\text{-independent} \quad (17)$$

$$(TLL')^P := TLL' - \partial^\mu TV_\mu L' - \partial^\mu TLV'_\mu + \partial^\mu \partial^\nu TV_\mu V'_\nu, \quad d := d_e + d_{e'} \quad (18)$$

It can be shown that this is automatically fulfilled in massive spinor QED, but in scalar massive QED this turns into a normalization requirement which *induces* the second order  $A \cdot A \varphi^* \varphi$  term which is well-known from gauge theory [14].

The "magic" of  $L, V_\mu$  pairs with (16) is that on the one hand they permit to use the lower short distance dimension of string-local fields and in this way lower the power-counting bound of renormalizability, but they also guaranty the  $e$ -independence of the S-matrix since the derivative contributions in (18), by which the point-local density  $(TLL')^P$  differ from their string-local counterpart, are disposed off in the adiabatic limit

$$S^{(2)} = S_S^{(2)} = S_P^{(2)} \quad (19)$$

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<sup>7</sup>Its point-local counterpart  $A_\mu \rightarrow A_\mu^P$  has  $d_{sd}(L^P) = 5$  and hence nonrenormalizable. Note that unless specifically stated all our fields act in a Hilbert space.

The extension of this observation to the construction of time-ordered correlation functions of string-local interacting fields leads to their independence from *inner*  $e$ 's i.e. from string directions of inner propagators<sup>8</sup>. A string-local interaction density can only maintain higher order string-localization if it is the  $L$  of a  $L, V_\mu$  pair.

Since the integration over inner  $x$ 's of strings  $x + \mathbb{R}_+ e$  would lead to a total delocalization; the gain in renormalizability by replacing  $L^P$  by  $L$  would be worthless since such an ansatz does not qualify as a model of QFT. As the BRST gauge invariance of the S-matrix  $\mathfrak{s}S = 0$  is not a property of a single contribution but rather that of a sum of many, the  $e$ -independence results from on-shell cancellations between different contributions in the same order. The  $L, V_\mu$  (or  $L, Q_\mu$ ) pair property (16) is a necessary condition for this to happen; it permits to sail between Scilla of nonrenormalizability and Charybdis of total delocalization. The main point of the present work will be that such a path is not possible in the presence of WS fields in  $L$ .

There is another important physical aspect of the  $L, V_\mu$  pair. In contrast to the gauge formalism, the scalar  $\phi$  is not a nuisance of the formalism (as the Stückelberg  $\phi^K$  and the ghosts). It is a physical field of the Hilbert space formulation and as such an indispensable object of massive QED (as the abelian Higgs model not realized in nature). This is reminiscent of the presence of the bosonic Cooper pairs in the BCS description of superconductivity; without their presence it is not possible to obtain the short ranged vector potentials in London's first phenomenological argument. They are not additional degrees of freedom but result from rearrangements of existing condensed matter degrees of freedom at low temperature.

Before we address the problem of WS matter, it is instructive to look at a slightly more involved situation in which the string-local  $\phi$ -escort of  $A_\mu$  already enters the model-defining first order interaction density. Such an example is provided by an interaction of  $A_\mu$  with a Hermitian (instead of complex) scalar field  $H$ . As known from point-local interaction it is sufficient to specify the field content of a model (which includes the masses of the participating free fields). Starting with a particular interaction density of lowest short distance dimension, the model completes itself by generating additional local renormalization counterterms with new coupling parameters (in case of string-local free fields also *induced* terms as  $A \cdot A\varphi^*\varphi$  without new couplings) so that the result is independent on the starting interaction density and only depends on the field content and imposed inner symmetries.

For an  $/A, H$  field content the point-local interaction with the lowest short distance dimension is  $L^P = mA^P \cdot A^P H$ . Writing it in the form of a string-local  $L, V_\mu$  pair one obtains:

$$L = m \left\{ A \cdot (AH + \phi \overleftrightarrow{\partial} H) - \frac{m_H^2}{2} \phi^2 H \right\}, \quad V_\mu = m \left\{ A_\mu \phi H + \frac{1}{2} \phi^2 \overleftrightarrow{\partial}_\mu H \right\} \quad (20)$$

$$L - \partial V = L^P = mA^P \cdot A^P H, \quad d(L - \partial V) = 0$$

In this case the on-shell  $e$ -independence requirement (17) of the tree contribution to second and third leads to a much richer collection of induced terms than in the case of scalar massive QED. Whereas in the latter case the  $e$ -independence of the second order S-matrix induces only the  $A \cdot A\varphi^*\varphi$  term, the induction in case of an interaction  $A \cdot AH$  with a Hermitian field interaction leads to  $H^4, \phi^4, H^2\phi^2$  terms which, if combined together, take the form of an induced (not postulated) Mexican hat potential [14].

The resulting model is the abelian Higgs model. Usually this model is not derived from BRST gauge invariance of the S-matrix  $\mathfrak{s}S = 0$  [20] or the  $d(L - \partial V) = 0$  condition for a  $L, V_\mu$  pair, but rather from imposing an off-shell gauge breaking prescription (the Higgs mechanism). These are different prescriptions which lead to the same second order result, but the correct physical interpretation is the one based on the  $e$ -independence of the S-matrix since this follows directly from the physical principles; the broken Mexican hat potential has not to be postulated but is a consequence of the Hilbert space formulation of

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<sup>8</sup>This will be the subject of forthcoming work by Jens Mund.

causal localization principles. In a theory where all fields in the interaction density become string-local there is also no symmetry which can be spontaneously broken.

Important is not the recipe by which one obtained one model from another but the intrinsic physical properties and they show that the Higgs model of an interaction of a massive vector meson interacting with a Hermitian field is the screened charge  $Q = 0$  of the identically conserved Maxwell current  $j_\mu = \partial^\nu F_{\mu\nu}$  of a massive vector meson [14] and not the conserved current of a SSB ( $\partial^\mu j_\mu = 0$  and  $Q = \int j_0 = \infty$  is the definition of SSB). To obtain a lesser known model (the interaction of a massive vector potential with a Hermitian scalar field) from a better known one (scalar QED) by a simple trick (adding a Mexican hat potential + an adjusting shift in field space) as long as one does not confuse such a recipe with a physical property of the resulting  $A$ - $H$  interaction.

QFT is certainly a fundamental theory, but it contains no information about the origin of masses of those particles which are associated to the "elementary" fields in terms of which one defines a model<sup>9</sup>. The first order string-local interaction density may however lead to *induced couplings* which relate mass ratios to coupling strengths.

The vector mesons of massive QED does not need the coupling to  $H$ 's, but *massive QCD, or merely self-interacting massive  $Y$ - $M$  fields, requires their presence*. The  $L, V_\mu$  pairs setting for such models satisfies the condition of  $e$ -independence. But now there is a new phenomenon which was absent in massive QED: *self-interacting massive vector mesons lead to a second order induced interaction with a  $d_{sd}^{int} = 5$  contribution. This is presently the only known QFT model for which the first order power-counting bound does not prevent a second order violation*. The solution within the  $L, V_\mu$  setting is to enlarge the model by a coupling to an additional field which leads to second order compensations of renormalizability-violating terms.

Hence the nonabelian massive vector mesons which improved the short distance behavior of the 4-Fermi coupling by transmitting the forces between the fermions did not, as originally expected, maintain renormalizability in second order. The solution was to compensate the renormalization-violating  $d_{sd} = 5$  second order contribution by a similar contribution from an added first order nonabelian  $A$ - $H$  interaction with an additional Hermitian  $H$  field. Fortunately its second order iteration, different from its abelian counterpart, contains a  $d_{sd} = 5$  contribution which cancels the incriminated contribution from the second order self-interaction. The upholding of renormalization and the avoidance of higher order complete delocalization attributes a fundamental property to the  $H$  particle in particular since this compensation can only be achieved by a Hermitian scalar field and the  $A$ - $H$  coupling turns out to be unique in case of only one  $H$  [16]. Deriving the total interaction via the SSB prescription would overlook the compensation mechanism.

The guiding idea of higher order compensations between contributions from different spins is reminiscent of how one pictures short distance compensations between different spin contributions in supersymmetric models. The difference is that in the present case the short distance improvements are the *raison d'être* for such an extension, whereas SUSY was originally invented for other reasons.

The  $L, V_\mu$  mechanism to preserve higher order renormalization and string-localization (avoidance of higher order total delocalization) may have interesting extensions to higher spins. In that case the short distance dimensions of the operators  $L$  remains at  $d_{sd}^{int} \leq 4$  but the dimensions of the  $V_\mu$  in (16) will be  $d_{sd}(V_\mu) > 5$ . What one needs are contribution from couplings to fields of lower spin fields with lead to renormalizability-preserving compensations in higher orders. The simplest model realization for such an idea would start from  $s = 2$   $g_{\mu\nu}$ -potentials interacting with lower  $s = 1$  and  $s = 0$  potentials. The string-local setting is still in its infancy, and the previous  $s = 1$  example of a compensation shows that such constructions are not easy.

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<sup>9</sup>Lattice model calculations suggest that masses associated to composite fields (bound state) are determined in terms of those of the fundamental fields and their couplings.

We now return to the main problem of the present work: interactions involving zero mass string-local fields. In the presence of a mass gap and asymptotic completeness<sup>10</sup> the conceptual situation is characteristic for particle theory: the quantum fields act in a Wigner-Fock Hilbert space of particles. The main problem is to construct  $L, V_\mu$  pairs in terms of massive  $s > 1$  tensor potentials and to study the massless limit of the correlation functions.

The breakdown of the particle structure of the Hilbert space and of the S-matrix is intimately related to the appearance of infrared problems whose spacetime interpretation is one of the still unsettled conceptual problems of QFT. In order to avoid misunderstandings, there are very successful momentum space description for photon-inclusive cross sections of collision of charge-carrying particles which replace the non-existing S-matrix. But there are no *general* arguments why these cross sections remain gauge invariant (only shown in low orders) in the absence of a gauge invariant S-matrix, nor is there a spacetime understanding in what way logarithmic on-shell infrared divergencies are related to the vanishing of the LSZ scattering limit for charge-carrying particles (the "softening" of mass-shell delta functions [32]).

The present string-local setting places the infrared problems of interacting massless vector potentials into a new light. It shows that these problems are not the result of shortcomings of the chosen formalism but rather reflect a profound conceptual change. The perturbative  $L, V$  operator formalism in Wigner-Fock space breaks down (the  $V$  diverges in the massless limit) and the only objects which can be expected to have a smooth massless limit are the *positivity preserving vacuum expectation values of string-local fields*. According to Wightman's reconstruction theorem they suffice to reconstruct operators acting in a new Hilbert space which is not a Wigner Fock space. There is no perturbative QFT formalism which permits to construct QED directly; in this respect the gauge theoretic description which through its point-local gauge dependent matter fields simulates a zero mass particle structure in Krein space which has no counterpart in the physical Hilbert space.

Actually the picture from gauge theory may be asymptotically correct for short distances i.e. the beta function of the Callan-Symanzik equation of the gauge formulation in Krein space may be the same as that in the SLFT Hilbert space description, assuming that a string-local formulation has an associated  $e$ -independent Callan-Symanzik equation. The correct counterparts to the long range Coulomb potentials in quantum mechanics are the zero mass limits of string-local charge-carrying fields.

## 4 Impossibility of perturbative interactions of infinite spin matter

In the previous section we posed the problem of perturbative interactions of higher spin  $s > 1$  fields. The conclusion was that in this case the the construction of first order interaction string-local interaction densities within  $d_{sd}^{int} \leq 1$  by itself is not a problem; but unless they are part of an  $e$ -independent  $L, V$  pair obeying  $d_e(L - \partial V) = 0$  they do not lead to a physical S-matrix and string-local correlation functions. In both cases there can be no dependence on inner  $e$ 's (the  $e$ 's of the propagators over whose  $x$ 's one integrates) and the  $L, V$  pair requirement together with the power-counting bound for  $L$  is a necessary condition for such an inner  $e$ -compensations between different contributions to a perturbative order<sup>11</sup>.

It may happen that the second order  $e$ -independence cannot be fulfilled without extending the model. As explained in the previous section this does not occur for massive spinor or scalar QED nor for the linear coupling of a massive vector meson to a Hermitian field (the abelian Higgs model) but it does happen

<sup>10</sup>In a perturbative setting asymptotic completeness is fulfilled since the Hilbert space is the Wigner-Fock space of the elementary free fields.

<sup>11</sup>This is similar to gauge invariance except that gauge invariance does not provide a point-local analog of physical string-dependent fields.

for massive QCD and just self-interacting massive vector mesons where a second order induced  $d_{sd} = 5$  coupling appears. As explained this situation can only be saved by including a compensating nonabelian  $A$ - $H$  coupling. So the power-counting bound is preserved and string-localization is maintained. This compensation mechanism is the theoretical basis for the perturbative consistency of the standard model.

For higher spins  $s \geq 2$  the prize for maintaining the power-counting bound for  $L$  is that  $V_\mu$  carries the high dimension  $d_{sd}(V_\mu) = 4 + (s - 1)$  which leads to induced second order power-counting violating terms. Again only an extension by couplings to lower spin fields and the hope for compensations can save the situation. Whether there are such higher spin cases will remain a problem for future investigations.

WS spin fields constitute an isolated class of massless string-local fields. Unlike the construction of massless free string-local *tensor potentials* from massless limits of massive string-local correlation functions, it is not possible to present string-local WS fields as a massless *limit of covariant tensorial potentials* for  $m \rightarrow 0$  which is necessary for formulating a string-local renormalization theory. It is presently unclear whether string-local transcendental intertwiners as (12) and their associated string-local fields can be obtained as  $m \rightarrow 0$ ,  $s \rightarrow \infty$  limits with  $\kappa^2 = m^2 s(s + 1)$ . This intertwiner is a special case of the most general string-local transcendental WS intertwiners in [8].

The transcendental two-point function (13) which corresponds to this intertwiner (12) is well-behaved for short and long distance, so the problems of interactions do not arise from renormalizability-threatening ultraviolet problems of free fields. Rather they originate from the breakdown of higher order string-localization which leads to a total delocalization; such candidates do not define interacting models of QFT.

Indeed any  $L$  of the form  $L = X\Phi$ , where  $\Phi$  is a WS field and  $X$  any product of standard free fields with  $L$  within the power-counting bound will lead to higher order interactions with internal  $\Phi$ -propagators whose integration over internal  $x$ 's will spread the string-like localization  $x + \mathbb{R}_+ e$  over all of spacetime. The necessary prerequisite for avoiding such a spread, namely the existence of a  $V_\mu$  with  $d_e(L - \partial V) = 0$  cannot be implemented i.e. WS cannot perturbatively interact with normal Wigner matter.

This is consistent with the structural result within the setting of algebraic QFT in [11]. There it was shown that the presence of WS in the field algebra leads to an unexpected property of the observable algebra of the combined model in case there is a nontrivial interaction of WS with normal matter. Namely the Reeh-Schlieder property, an important characteristic of compactly localized observable algebras, is necessarily violated. One way out is a tensor product between the WS and normal QFT matter (ignoring the coupling of WS to classic gravity); this agrees with the negative outcome of our perturbative attempt. Although *WS reacts to gravitational forces, it remains inert (forms a tensor product) with respect to normal matter*.

The gravitational coupling of a point-local scalar field uses the conserved traceless tensor as a source in the Einstein-Hilbert equation. The simplest conserved expression has the same form as for  $m = 0$  point-local fields

$$T_{\mu\nu} = \partial_\mu \varphi \partial_\nu \varphi - \frac{1}{2} g_{\mu\nu} \partial_\kappa \varphi \partial^\kappa \varphi \quad (21)$$

The string-localization of the WS free field  $\varphi(x, e)$  does not affect the validity of the Klein-Gordon equation, hence the tensor is conserved; however the corresponding candidates for the conserved charges are not the generators of the Poincaré group<sup>12</sup>.

For the formal consistency as a conserved source of the Einstein-Hilbert equations the conservation law suffices. There remains however the problem whether a conserved energy-momentum tensor whose conserved global "charges" are the generators of the Poincaré transformations exists. This is even a problem of massive  $s \geq 2$  tensor potentials. The candidate for such an energy-momentum tensor would

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<sup>12</sup>Free fields are known to admit infinitely many conservation laws.

be the Wick-ordered product

$$T_{\mu\nu} \simeq as \partial_\mu A_{\lambda_1 \dots \lambda_n} \partial_\nu A^{\lambda_1 \dots \lambda_n} + g_{\mu\nu} T \quad (22)$$

where the string-local  $A$ -tensor (10) is symmetric and the antisymmetrization  $as$  refers to the  $\mu$  respectively  $\nu$  with the symmetric  $\lambda$  tensor indices. The additional bilinear  $g_{\mu\nu}$  contribution must be such that  $T_{\mu\nu}$  is trace- and divergence-less.

If we were to use the point-local  $A^P$ -tensor we would obtain an expression whose short distance dimension is (apart from the  $s = 1$  case, where the expression coincides with the well-known energy-momentum tensor in terms of the field strength  $F_{\mu\nu}$ ), too large for an energy-momentum tensor with  $d_{sd} = 4$ . However for the *string-local*  $A$ -tensor in (22) the short distance dimension and the dimension in mass units coalesce. Both tensors are different by derivatives of escort fields (10).

Only if the contribution of the escort fields drops out in the global limit of the would be Poincaré charges, the string-local  $d_{sd} = 4$  energy-momentum tensor will be the correct one. For this one would have to prove that the escort fields do not contribute to the large volume limit which represent the global charges. Whereas they do not contribute to the adiabatic limit (the fundamental property for the existence of renormalizable  $L, V_\mu$  pairs), it is not obvious that the global charges will be  $e$ -independent. The large volume limit of charges from conserved currents is a quite delicate issue [17].

An argument in favor of a local representation of the Poincaré group comes from modular localization theory. As in point-local QFTs it is possible to construct unitary operators localized in a compact spacetime region  $\mathcal{O}$  which for sufficiently small transformation parameters implement the correct Poincaré transformations on operators localized in a suitably chosen smaller region inside  $\mathcal{O}$  ([17] chapter V), one expects that in string-local QFTs one should have a corresponding situation for  $\mathcal{O}$  replaced by a spacelike cone  $\mathcal{C}$ . The "split property" for interacting fields on which these arguments are based certainly hold for free fields.

Whether conserved string-local energy momentum tensors (22) with  $e$ -independent Poincaré charges exist for  $s \geq 2$  is a question of great relevance. It is part of a general problem of constructing  $e$ -independent charges from string-local currents of low short distance dimensions in higher spin QFT for which the point-local basic free fields have high short distance dimensions  $d_{sd} = s + 1$ . This issue should be clarified before one attempts to construct a WS energy-momentum tensor in terms of a limiting process  $m \rightarrow 0$ ,  $s \rightarrow \infty$  at fixed Pauli-Lubanski invariant  $\kappa$ . We hope to return to this problem in a future publication.

The principle reason for the inert behavior of WS with respect to normal matter is that the WS representations is totally isolated from the other classes which cover the standard matter. There are no massive string-local finite spin covariant tensor fields whose vacuum expectation values converge to those of WS fields in the infinite spin limit at fixed Pauli-Lubanski invariant  $\kappa$ . Their remains another conceptual problem with the coupling of WS to gravity. As a positive energy representation WS couples in the classical sense of Einstein's dictum that all positive energy representations couple to classical gravity.

A clarification of the issue of an infinite spin energy-momentum tensor would open the possibility to do calculations within the setting of the Einstein-Hilbert equation. Further astrophysical observations may reveal that the relation between WS and quantum aspects of gravity are much stronger than envisaged in present ideas about the nature of  $s = 2$  quantum gravity in which the infinite spin WS and gravity remain separate states of massless matter.

## 5 Concluding remarks

As all positive energy matter, the Wigner stuff couples to gravity and for this reason enters the galactic gravitational balance. This together with its inert behavior with respect to ordinary matter makes it an interesting candidate for dark matter in the sense of astrophysical observations apart from the fact that

as a factor in a tensor product with normal matter which is only bridged by classical gravity it may be a bit "too perfect". Such world of dark quantum matter which does not communicate with normal quantum matter may be hard to digest from a philosophical point of view, although (to the author) it appears less bizarre than the proposal of multi-verses.

But there remain valid astrophysical objections and questions as: how did noncompact localized matter get into our universe and what was its role in the formation of ordinary matter after the big bang? Here high temperature phase transitions may have played a role in converting a high temperature compact localized and more reactive form of infinite spin matter into its present noncompact inert vacuum representation.

Unlike other dark matter candidates (WIMPS,...), the Wigner stuff has not been invented to explain dark matter; as a rather big class being characterized in terms of the continuous invariant  $\kappa$ , it made its debut already in Wigner's 1939 paper which was written in the same decade in which Zwicky discovered dark matter from his analysis of galactic observations. What was missing for nearly seven decades was an understanding of its unexpected causal localization properties on which the present ideas in this paper are based.

Any laboratory observation of a new form of matter which can only be explained by identifying it with the ubiquitous galactic dark matter will invalidate the present proposal. But even if this happens the present work may not have been in vain since the identification of its intrinsic noncompact localization [11] [8] WS led in turn to a better understanding of the important role of the interplay between Hilbert space positivity and string-local causal localization properties of interacting quantum fields of higher spin [8] [9] [14] [16].

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