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On the two-slit interference experiment with electrons: a theoretical prediction of Bohm-de Broglie quantum mechanics different from the prediction of usual Copenhagen quantum mechanics

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The Bohm-de Broglie quantum mechanics has made possible to calculate the trajectories of electrons in a typical double-slit experiment[C. Philippidis et al., Il Nuovo Cimento, 52 B, 15-28 (1979)]. The trajectories do not correspond to an uniform movement but to an accelerated one. The acceleration is caused by the quantum potential. From the quantum theoretical point of view, the accelerated electrons should, with a certain probability, emit photons during its movement from the slits to the screen. According to the Copenhagen interpretation we found that this quantity is strictly zero because the electron moves as a free particle along its path after it leaves the slit and before reach the screen. Then, there is no emission of photons. On the other hand, by using the Bohm-de Broglie (BdB) approach we calculate the emission power, giving a general formula and which results, for a concrete real experiment, in a very tiny but not a vanishing value. We give an idea of the type of spectrum that could be measured. Therefore we have shown that the theoretical predictions of the usual quantum mechanics and that of Bohm-de Broglie's interpretation are different. An experiment could determine the correct prediction.

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I. INTRODUCTION

In Feynman, Leighton and Sands 's words [1] the interference experiment with electrons "'...has in it the heart of quantum mechanics. In reality, it contains the *only* mystery."' The formation of the interference pattern has been demonstrated in several experiments, among them Jönsson (1961)[2], Tonomura et. al (1989)[3]. At the same time the theoretical explanation indicated that "'The electrons arrive in lumps, like particles, and the probability of arrival of these lumps is distributed like the distribution of intensity of a wave. It is in this sense that an electron behaves sometimes like a particle and sometimes like a wave."[1], or "'In quantum mechanics there is not such concept as the path of a particle."'[4]. On the other hand the trajectories of the electrons in an typical two-slit interference experiment were computed and plotted in the framework of the BdB approach to quantum mechanics [5]. This is a version of quantum mechanics that reproduces all the experimental results explained by usual quantum mechanics [6][7]. Since the introduction of this version of quantum mechanics, people have been looking for a way to rule it out through some experiment they can not explain. This has not been possible so far. In this work we are going to propose an experiment for which the theoretical prediction of the two versions of the quantum mechanics, Copenhagen and Bohm-deBroglie, are different. It will then suffice to measure to choose the one whose prediction is correct. This experiment is neither more nor less than the typical interference experiment of electrons for which we will show that the usual interpretation predicts that they do not radiate on their way to the screen and instead the interpretation of BdB predicts that electrons, being accelerated, radiate with a very small power, on its way to the screen. When the present work was ready to be submitted to be considered for publication, we learned of a very interesting preprint by Pisin Chen, more than twenty years ago in which a proposal similar to ours had already been discussed [8]. In that preprint an analytical model of the quantum potential of the experiment of the two slits with electrons was used and a numerical study was also carried out. It was concluded, like us in the present work, that the electrons must emit radiation. We can say that this preprint and the present work, in a certain way, complement each other since, in our section III, we make a graphic study in scale, somewhat handmade and in Chen's work more general methods were used. However, in addition to the fact that Chen's prediction is mainly in the visible range, a range different from that predicted in the present work, namely radio waves[1], the same author in a more recent preprint concludes that this radiation does not really exist [9].Therefore, we think that our proposal is of additional interest since, using different methods, we affirm that the BdB view of quantum mechanics says that this radiation should exist.

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[1] However, we note that the width of the slits used in Chen's work is different.

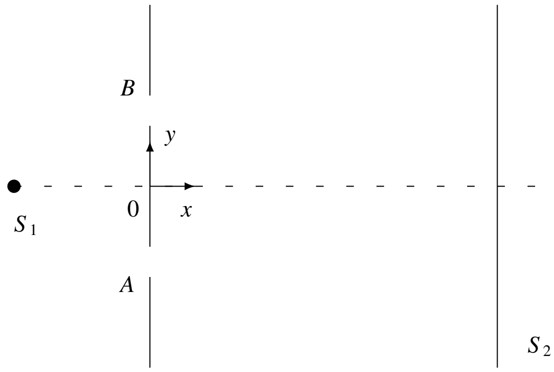

FIG. 1. Two-slit interference experiment for electrons.

This letter is organized as follows: In section II we present the typical experiment and we made the calculations following usual quantum mechanics using an elementary pedestrian approach to quantum electrodynamics. In section III we make the computations according to the BdB approach giving first an exact formula for the emission power by electrons and then an estimate for its value in the case of two real experimental possibilities. In section IV we say two words about polarization of the emitted radiation and Section V is for discussion and conclusions. Some calculations have been placed in appendices, in order not to separate the reader from the main line of reasoning.

II. ELEMENTARY COMPUTATION OF THE PROBABILITY OF EMISSION OF A PHOTON BY AN ACCELERATED ELECTRON, ACCORDING TO THE COPENHAGEN INTERPRETATION

Lets consider an usual two-slit experiment given by an electron source S_1, two slits A and B and a screen S_2.

We adopt a co-ordinate system with origin at O as indicated in the Fig. 1, with the centers of the slits having co-ordinates (O, Y) and $(O, -Y)$. (same convention that as [5]). After go through and coming out the slits an electron becomes free, i.e. the potential acting on it is zero

In the experiment presented by Jönsson in [2] which is the same later studied in [5], the kinetic energy of a typical electron is $45\,keV$. This represents approximately 9% of its rest mass, $511keV$, and this mean a non-relativistic case [5]. Then the hamiltonian operator of the electron is given by[2].

$$H_e = \frac{\boldsymbol{p}^2}{2m} \tag{1}$$

If we consider that a photon can be emitted, these are characterized by the potential vector $\boldsymbol{A}(\boldsymbol{r})$ and then the Hamiltonian of the total electron + photon system is

$$H = \frac{1}{2m}(\boldsymbol{p} - \frac{e}{c}A)^2 + \frac{1}{8\pi}\int dx^3 (E^2 + B^2) \tag{2}$$

that can be written as

$$H = H_e + H_{rad} + H_I \tag{3}$$

where

$$H_I = -\frac{e}{mc}\boldsymbol{p}\cdot\boldsymbol{A} + \frac{e^2}{2mc^2}A^2 \tag{4}$$

$$H_{rad} = \frac{1}{8\pi}\int dx^3 (E^2 + B^2) \tag{5}$$

The term H_I will be treated as a perturbation. The Hamiltonian without perturbation

$$H = H_e + H_{rad} \tag{6}$$

has as eigenvectors

$$|e + radiation\rangle = |\boldsymbol{p}\rangle\,|...n_{\boldsymbol{k}\sigma}..\rangle_{rad}\ . \tag{7}$$

The interaction hamiltonian H_I induce transitions between this states, and the transition probability per unit time is given by the Fermi 'Golden Rule" as:

$$\frac{prob}{time} = \frac{2\pi}{\hbar}\left|M_{fi}\right|^2 \delta(E_f - E_i) \tag{8}$$

where

$$\left|M_{fi}\right| = \langle f|\,H_I\,|i\rangle + \sum \frac{\langle f|\,H_I\,|n\rangle\,\langle n|\,H_I\,|i\rangle}{E_i - E_f + i\eta} + \frac{\langle f|\,H_I\,|n\rangle\,\langle n|\,H_I\,|m\rangle\,\langle m|\,H_I\,|i\rangle}{(E_i - E_n + i\eta)(E_i - E_m + i\eta)} + ... \tag{9}$$

Now we write H_I as

$$H_I = H' + H''\,, \tag{10}$$

where

$$H' \equiv -\frac{e}{mc}\boldsymbol{p}\cdot\boldsymbol{A} \tag{11}$$

$$H'' \equiv \frac{e^2}{2mc^2}\boldsymbol{A}^2 \tag{12}$$

and substituting the expansion in normal modes for $\boldsymbol{A}$

$$A(x,t) = \sum_{k,\sigma} \sqrt{\frac{2\pi\hbar c^2}{\Omega\omega_k}} \boldsymbol{u}_{\boldsymbol{k}\sigma} \left[a_{\boldsymbol{k}\sigma}(t) e^{i\boldsymbol{k}\boldsymbol{r}} + a^{+}_{\boldsymbol{k}\sigma}(t) e^{-i\boldsymbol{k}\boldsymbol{r}} \right]$$

being

$$a_{\boldsymbol{k}\sigma}(t) = a_{\boldsymbol{k}\sigma}(0) e^{-i\omega_k t} \tag{13}$$

$$a^{+}_{\boldsymbol{k}\sigma}(t) = a^{+}_{\boldsymbol{k}\sigma}(0) e^{i\omega_k t} \tag{14}$$

the destruction and creation operators, and $\Omega \equiv$ volume of the box where the electromagnetic field is quantized

we obtain:

$$H' = -\frac{e}{mc} \boldsymbol{p} \sum_{\boldsymbol{k}\sigma} \sqrt{\frac{2\pi\hbar c^2}{\Omega\omega_k}} \boldsymbol{u}_{\boldsymbol{k}\sigma} \left[a_{\boldsymbol{k}\sigma} e^{i\boldsymbol{k}\boldsymbol{r}} + a^{+}_{\boldsymbol{k}\sigma} e^{-i\boldsymbol{k}\boldsymbol{r}} \right]$$

$$H'' = \frac{e^2}{2mc^2} \sum_{\boldsymbol{k}\sigma} \sum_{\boldsymbol{k}'\sigma'} \left(\frac{2\pi\hbar c^2}{\Omega}\right) \frac{1}{\sqrt{\omega_k \omega'_k}} \boldsymbol{u}_{\boldsymbol{k}\sigma} \boldsymbol{u}_{\boldsymbol{k}'\sigma'} \times$$
$$\left[a_{\boldsymbol{k}\sigma} a_{\boldsymbol{k}'\sigma'} e^{i(\boldsymbol{k}+\boldsymbol{k}')\boldsymbol{r}} + a_{\boldsymbol{k}\sigma} a^{+}_{\boldsymbol{k}'\sigma'} e^{i(\boldsymbol{k}-\boldsymbol{k}')\boldsymbol{r}} + a^{+}_{\boldsymbol{k}\sigma} a_{\boldsymbol{k}'\sigma'} e^{i(-\boldsymbol{k}+\boldsymbol{k}')\boldsymbol{r}} + a^{+}_{\boldsymbol{k}\sigma} a^{+}_{\boldsymbol{k}'\sigma'} e^{i(-\boldsymbol{k}-\boldsymbol{k}')\boldsymbol{r}} \right]$$

To the first order in the perturbation we see that H' induce transitions in which the number of photons changes in one unity (i.e ± 1), since one and only one creation or destruction operator appear in each term of it. In the same way we see that H'' induces changes in which two photons are emitted or two are absorbed or one is emitted and another is absorbed.

Let's consider the electron with initial state $|p_a\rangle$ and final state $|p_b\rangle$. We are going to analyze the emission of one photon with wave vector $\boldsymbol{k}$ and polarization σ. We write for the initial and final states:

$$|i\rangle = |p_a\rangle \, |...n_{k\sigma}...\rangle_{rad} \tag{15}$$

$$|f\rangle = |p_b\rangle \, |...n_{k\sigma} + 1...\rangle_{rad} \tag{16}$$

Transitions between this states can only be induced by H' in the first order contribution to M_{fi}.

We have

$$\langle f| H' |i\rangle = -\frac{e}{mc} \sqrt{\frac{2\pi\hbar c^2}{\Omega\omega_k}} \langle p_b| \boldsymbol{p}.\boldsymbol{u}_{\boldsymbol{k}\sigma} e^{-i\boldsymbol{k}\boldsymbol{r}} |p_a\rangle \sqrt{n_{\boldsymbol{k}} + 1} \tag{17}$$

then, from eq. (8)

$$\frac{prob}{time} = \frac{4\pi^2 e^2}{m^2\Omega\omega_k} (n_{\boldsymbol{k}} + 1) \left| \langle p_b| \boldsymbol{p}.\boldsymbol{u}_{\boldsymbol{k}\sigma} e^{-i\boldsymbol{k}\boldsymbol{r}} |p_a\rangle \right|^2 \delta(E_b - E_a + \hbar\omega_k) \tag{18}$$

The case $n_k = 0$ correspond to the situation in which there is no photons a priori before the emission, in the final state. In order to calculate the lifetime of the state $|p_a\rangle$ against the emission of a photon it is necessary to sum over all the possible values of $\boldsymbol{k}$ and σ that the emitted photon can have. In summing over the polarizations we choose $\boldsymbol{u}_{\boldsymbol{k}1}$ and $\boldsymbol{u}_{\boldsymbol{k}2}$ as in Fig. II, then

$$\sum_{\sigma=1,2} \left| \langle p_b| \boldsymbol{p}.\boldsymbol{u}_{\boldsymbol{k}\sigma} e^{-i\boldsymbol{k}\boldsymbol{r}} |p_a\rangle \right|^2 = \left| \langle p_b| \boldsymbol{p} e^{-i\boldsymbol{k}\boldsymbol{r}} |p_a\rangle \right|^2 \sin^2\theta \tag{19}$$

Now it is a reasonable approximation to say that the emitted photon is of very low energy, i.e. it have a very long wavelength compared with the characteristic dimensions of the experiment which means $\exp -i\boldsymbol{k}\boldsymbol{r} \cong 1$. Substituting in (18), taking into account that in summing on all the possible states the probability in the numerator becomes 1 and using the prescription

$$\sum_k \longrightarrow \frac{\Omega}{(2\pi)^3} \int d^3k \, , \tag{20}$$

[2] In this section we follow the lines and notation of [10] Ch. 3. but keep in mind that here we have an electron instead of an atom.

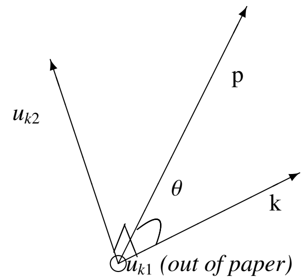

FIG. 2. Polarization of photons for the sum in Eq. (19). $\boldsymbol{u_{k1}} \perp \boldsymbol{u_{k2}}$ and both perpendicular to the direction of propagation of the photon $\boldsymbol{k}$

for $\Omega \to \infty$ (the volume of the box going to infinity) we obtain:

$$\left(\frac{1}{\tau}\right)_{a\to b} = \frac{e^2}{2m^2\pi}\int d^3k\frac{1}{\omega_k}\left|\langle p_b|\,\boldsymbol{p}\,|p_a\rangle\right|^2 \sin^2\theta\delta(E_b - E_a + \hbar\omega_k) \tag{21}$$

Now we use spherical coordinates in the k-space with the z-axis in the $\langle p_b|\,\boldsymbol{p}\,|p_a\rangle$ direction, then:

$$d^3k = k^2 dk \sin\theta d\theta d\phi \tag{22}$$

and using $k = \frac{w_k}{c}$, we have

$$d^3k = \frac{w_k^2}{c^3}dw_k \sin\theta d\theta d\phi \tag{23}$$

which substituting in (21) and integrating, allow us to write:

$$\left(\frac{1}{\tau}\right)_{a\to b} = \frac{4e^2}{3m^2c^3\hbar}\omega_{ab}\left|\langle p_b|\,\boldsymbol{p}\,|p_a\rangle\right|^2 \tag{24}$$

were $\omega_{ab} = \frac{(E_a - E_b)}{\hbar}$ is the frequency of the emitted photon.

Using that $\boldsymbol{p} = m\dot{\boldsymbol{r}}$ (non relativistic case) we can re-write the bracket in (24) in the following form [3]:

$$\langle p_b|\,\boldsymbol{p}\,|p_a\rangle = \frac{im}{-\omega_{ba}}\langle p_b|\,\ddot{\boldsymbol{r}}\,|p_a\rangle \tag{25}$$

that substituted in (24) gives

$$\left(\frac{1}{\tau}\right)_{a\to b} = \frac{4e^2}{3c^3\hbar\omega_{ba}}\left|\langle p_b|\,\ddot{\boldsymbol{r}}\,|p_a\rangle\right|^2 , \tag{26}$$

where we used that $\omega_{ab} = -\omega_{ba}$.

[3] See appendix A.

Being the energy of the emitted photon equal to $\hbar\omega_{ba}$, we have for the energy radiated per unit time (emission power)

$$\left(\frac{\hbar\omega_{ba}}{\tau}\right)_{a\to b} = \frac{4e^2}{3c^3}\left|\langle p_b|\,\ddot{\boldsymbol{r}}\,|p_a\rangle\right|^2 \tag{27}$$

that we write as

$$\left(\frac{dE}{dt}\right)_{a\to b} = \frac{4e^2}{3c^3}\left|\langle p_b|\,\ddot{\boldsymbol{r}}\,|p_a\rangle\right|^2 . \tag{28}$$

We see a great resemblance to the Larmor's formula for an accelerated electron of Classical electrodynamics.

We can write the last equation using the "'fine structure constant": $\alpha = \frac{e^2}{\hbar c}$ as

$$\left(\frac{dE}{dt}\right)_{a\to b} = \frac{4}{3}\frac{\alpha\hbar}{c^2}\left|\langle p_b|\,\ddot{\boldsymbol{r}}\,|p_a\rangle\right|^2 \tag{29}$$

Now we analyze the electron considered in our problem, which once abandoned the slits experiences no potential before striking the screen (i.e. $V(\boldsymbol{r}) = 0$ for $x_{slit} < x < x_{screen}$) . This mean that its hamiltonian is that of a free particle, Eq. (1), and then $\ddot{\boldsymbol{r}}$ vanish:

$$\ddot{\boldsymbol{r}} = \frac{i}{\hbar}[H_e, \dot{\boldsymbol{r}}] = \frac{i}{\hbar m}[H_e, \boldsymbol{p}] = \boldsymbol{0} . \tag{30}$$

Then, equation (29) gives:

$$\left(\frac{dE}{dt}\right)_{a\to b} = 0 , \tag{31}$$

i.e. a null value, which means the energy emitted per unit time is zero and we have no photon emitted[4]. This is the answer that gives the Copenhagen Interpretation of QM. The electrons goes from the slits to the screen without emission of photons in most of its travel, with a possible exception in two points which are the final point in the screen and the initial passing through the slits, point where the potential is not vanishing. Note that, following the founders Bohr, Heisenberg, Landau and others, we have not talked about "'trajectory"' in our deduction.

We followed an elementary computation in Q.E.D., i.e. we have not made use of the 2nd quantization formalism but, as it is well known, the answer to the problem must be the same with the only price to pay being the lost of manifestly covariant equations.

[4] In a sense we verified the Feynman's quote *[...] the electron cannot emit a photon and make a transition to a different electron state while traveling along a vacuum [11].*

III. PHOTON EMISSION IN THE BOHM-DE BROGLIE APPROACH

The two-slit interference experiment with electrons was studied in the framework of the BdB quantum mechanics in [5] where the bohmian trajectories were first calculated (Fig. 3). An interesting discussion of this experiment at the light of BdB quantum mechanics is given in [6]. The trajectory of an electron is affected by the quantum potential which is depicted in Fig. 4. The plot of the quantum potential shows, after the high spikes in the central region near the slits, a set of troughs and plateaus. An electron emerging for one slit can be first repelled by the central spikes and then moves practically uniformly (with a small component of velocity in the y-direction) until it encounters one of the troughs in Q. One can have an idea of the variation of Q in the y axis by plotting the cross section which is depicted at about 18 cm from the plane of the slits: we see a series of "'potential wells" (or valleys) corresponding to the troughs (Fig. 5). The electron "'fall"' in the potential well where is first accelerated with a strong force $\frac{-\partial Q}{\partial y}$ and then decelerated (the quantum potential for this experiment depends only on y, see [5], [6]). From the quantum theoretical point of view we can say that there exist a certain probability for the accelerated electron emit a photon and the formula obtained for the emission power, Eq. (29) can be used but now accepting that the electron follows a trajectory. It is possible to write this equation as (see Appendix B):

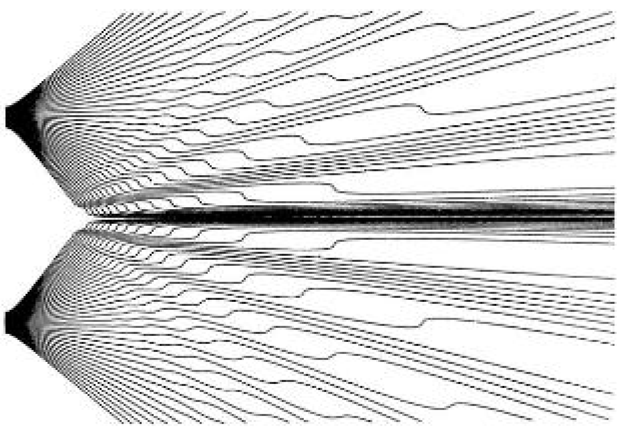

FIG. 3. Trajectories of the electrons in the two-slit interference experiment predicted by Bohm-de Broglie mechanics (Extracted from [5]).

$$\left(\frac{dE}{dt}\right)_{a\to b} = \frac{4}{3}\frac{\alpha\hbar}{c^2}\,(\ddot{\boldsymbol{r}})^2_{a\to b} \tag{32}$$

where $\boldsymbol{r}$ represents now the position of the electron.

Adapted to our goals, we have in a finite form:

$$\left(\frac{\Delta E}{\Delta t}\right) = \frac{4}{3}\frac{\alpha\hbar}{c^2}\left(\frac{\Delta \boldsymbol{v}}{\Delta t}\right)^2 \tag{33}$$

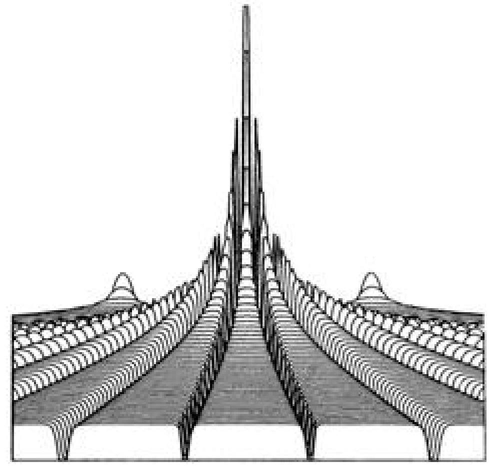

FIG. 4. The quantum potential for the two (gaussian) slits viewed from the screen S_2 (Extracted from [5]).

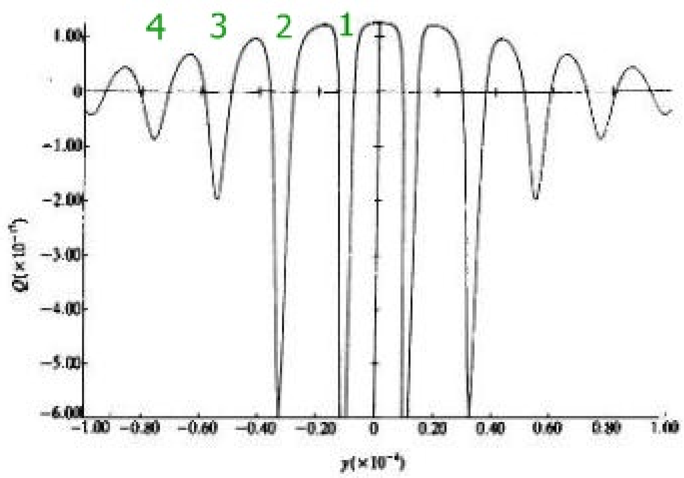

FIG. 5. A section of the quantum potential at 18cm from the slits S_2 (Extracted from [5]).

where $\boldsymbol{v} = \dot{\boldsymbol{r}}$ and we suppressed the sub-indices "a" and "b" .

We can estimate the last factor $\left|\frac{\Delta v}{\Delta t}\right|$ in (33) (or $\ddot{\boldsymbol{r}}$ in (32)) from the dynamical equation for the electron according to the BdB interpretation, which is a type of Newton's second law equation, in which the particle is subject to a quantum force $-\nabla Q$ in addition to the classical force $-\nabla V$. This equation reads as follows [7][6]:

$$\frac{d}{dt}(m\dot{\boldsymbol{r}}) = -\nabla(V+Q)|_{\boldsymbol{r}=\boldsymbol{r}(t)}\,. \tag{34}$$

In the experiment analyzed here we have $V = 0$ for the electron along the trajectory from the slits to the screen (excluding this extremal points).

Then

$$\frac{d}{dt}(m\dot{\boldsymbol{r}}) = -\nabla(Q)|_{\boldsymbol{r}=\boldsymbol{r}(t)}\,. \tag{35}$$

or

$$\frac{d}{dt}(\dot{\boldsymbol{r}}) = -\frac{1}{m}\nabla(Q)|_{\boldsymbol{r}=\boldsymbol{r}(t)}\,. \qquad (36)$$

which gives, after substitution in (32):

$$\left(\frac{dE}{dt}\right)_{a\to b} = \frac{4}{3}\frac{\alpha\hbar}{m^2c^2}\,(\nabla(Q))^2_{\boldsymbol{r}=\boldsymbol{r}(t)} \qquad (37)$$

This is the general answer that BdB approach gives for the emission power in a double slit experiment with electrons.

Note that in the work of Chen the Larmor formula of classical electrodynamics (Eq. (9) of [8]) is applied directly, which differs with respect to the formula (37) that we have found in a factor 2. It is noteworthy that we could also have considered the BdB interpretation of the electromagnetic field. In this sense, it is possible to show that in such a case the formula (32) is maintained. However, it may be of interest to give an ontological description of the radiation emission process studied, considering, instead of Fock states, i.e. the state (16) (which are equivalent to plane waves), non-stationary states given by packages or superpositions of Fock states with a certain function of weight, which allow to describe in a more realistic way the process. In this case, the weight function would be included in the formula of the emission power obtained. Bohm, in his 2nd article on "hidden variables" of 1952, developed the causal interpretation of the electromagnetic field and studied, in particular, the photoelectric and Compton processes from that point of view using non-stationary states [7]. A valuable report on the BdB interpretation of the electromagnetic field can be found in [12].

In order to continue the analysis with data from a concrete and real experiment we write the last one (following 33) as

$$\left(\frac{\Delta E}{\Delta t}\right) = \frac{4}{3}\frac{\alpha\hbar}{m^2c^2}\left(\frac{\Delta Q}{\Delta \boldsymbol{r}}\right)^2 \qquad (38)$$

It is possible estimate an approximate value for the gradient $|\nabla Q| \cong \left|\frac{\Delta Q}{\Delta y}\right|$ graphically from Fig. 5 when the electron enters each well (recall here Q depends only on y: $Q = Q(y)$). The maximum absolute value of Q is, for the Jönsson experiment, approximately equal to $10^{-4}eV$ [5][6]. From this we roughly estimate that for the 2nd well (counting from the symmetry center between the slits) we have a variation of $\frac{7}{16}\times 10^{-4}eV$ along a distance of $\frac{1}{7}\times 10^{-4}cm$ then:

$$\nabla Q \cong \frac{\Delta Q}{\Delta y} \cong \frac{\frac{7}{16}\times 10^{-4}eV}{\frac{1}{7}\times 10^{-4}cm} \cong 3.06\frac{eV}{cm} \qquad (39)$$

In other words, we approximate the curvilinear walls of each well shown in the Fig. 5 by straight walls, as it is shown in Fig. 6.

Substituting (39) in (38) and using the standard values:

$\hbar \cong 0.65\times 10^{-15}eV.s$

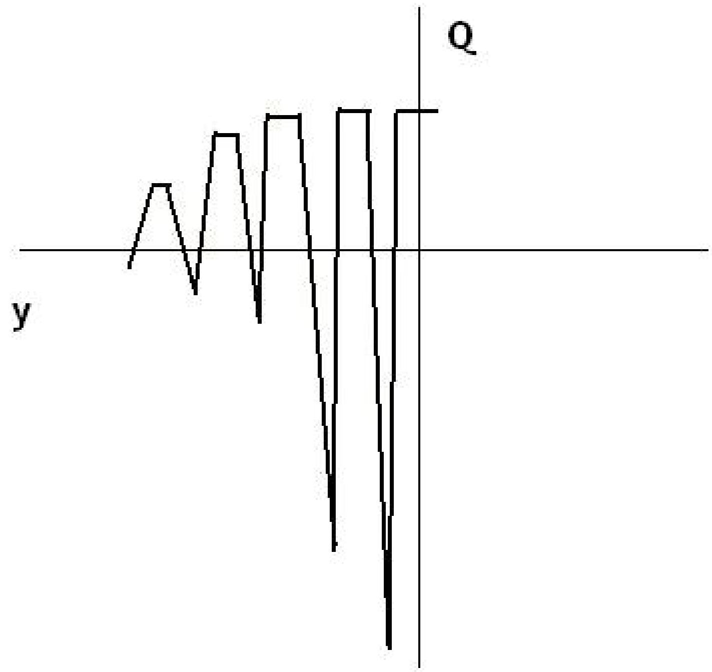

FIG. 6. Approximating the section of the quantum potential given in Fig. 5.

$mc^2 \cong 0.511\times 10^6 eV$
$c \cong 3\times 10^{10}\frac{cm}{s}$
$\alpha \cong \frac{1}{137}$

we obtain for the mean emission power:

$$P_2 \equiv \left(\frac{\Delta E}{\Delta t}\right) \cong 3.27\times 10^{-26}W\,. \qquad (40)$$

In the same way, for the 3rd well we have approximately $\frac{\Delta Q}{\Delta y} \cong \frac{350\times10^{-4}eV}{376\times10^{-4}cm} = 0.93\frac{eV}{cm}$ and the mean emission results in

$$P_3 \cong 3.02\times 10^{-27}W\,. \qquad (41)$$

For the 4th well we obtain $\frac{\Delta Q}{\Delta y} \cong 0.8\frac{eV}{cm}$ and the emission power is

$$P_4 \cong 3.27\times 10^{-26}W\,. \qquad (42)$$

Still for the first well, by extrapolating below the abscissa axis, we can obtain $\frac{\Delta Q}{\Delta y} \cong 9.66\frac{eV}{cm}$ and for the emission power

$$P_1 \cong 3.25\times 10^{-25}W\,. \qquad (43)$$

It is possible to have a crude idea for the frequency of some of the photons emitted. For that we estimate the time the electron takes to cross the well (this is the *collision time*, call it τ)[5]. We consider that the movement during the passage through the valley in the y axis occurs approximately with constant average accelerations (in the first half of the valley accelerated and in the second half decelerated) which can be obtained graphically. Furthermore we make another simplification which is to consider each valley as symmetrical along its central vertical axis.

Using for the initial y-velocity, in the border of the valley, the value from the experiment performed by Jönsson, $v_y = 1,5 \times 10^4 \frac{cm}{s}$ we found approximately for the 2nd valley (or well)

acceleration:

$$|a_2| \cong \frac{3.06}{m_e}\frac{eV}{cm} = 5.99 \times 10^{-6}\frac{c^2}{cm} = 5.39 \times 10^{15}\frac{cm}{s^2} \qquad (44)$$

collision time:

$$\tau_2 \cong 7.01 \times 10^{-11}\, s\,. \qquad (45)$$

Accepting that during this time a photon is emitted, by using the mean emission power given by Eq.(40) we have for the energy of the photon:

$$\begin{aligned} E_2 = P_2\tau_2 &= 3.27 \times 10^{-26} W \times 7.01 \times 10^{-11} s \\ &= 2.29 \times 10^{-36} j\,. \end{aligned} \qquad (46)$$

To this photon must correspond a frequency

$$\nu_2 = \frac{E_2}{h} = \frac{2.29 \times 10^{-36} j}{6.63 \times 10^{-34} j.s} = 3.45 \times 10^{-3}\, Hz\,. \qquad (47)$$

For the photons emitted as en electron cross the 3rd valley we found the collision time $\tau_3 \cong 1.02 \times 10^{-10} s$ and a frequency ν_3 approximately equal to:

$$\nu_3 \cong 4.66 \times 10^{-4}\, Hz\,. \qquad (48)$$

In the same way we obtain the collision time and a frequency for the 4th and still for the 1st valley:

$$\tau_4 \cong 1.09 \times 10^{-10}\, s\,, \qquad (49)$$

$$\nu_4 \cong 3.6 \times 10^{-4}\, Hz\,, \qquad (50)$$

$$\tau_1 \cong 2.8 \times 10^{-11}\, s\,, \qquad (51)$$

$$\nu_1 \cong 1.37 \times 10^{-2} Hz\,. \qquad (52)$$

Then, the electron irradiate soft photons (i.e. photons with small energies compared to the energy available in the experiment) and this is a key information because the emission of soft photons by accelerated electrons was already studied in [13]. We can take advantage of the results obtained there in order to estimate qualitatively the emission spectrum for all frequencies. It is important to note that the results presented by [13], in particular Eq. (15.2) "'holds quantum mechanically as well as classically"', in words of Jackson, page 709. In the case of non-relativistic collisions there is significant radiation for when the following condition is satisfied

$$\omega < \frac{1}{\tau}\,, \qquad (53)$$

where ω is the angular frequency, i.e $\omega = 2\pi\nu$ and τ is the collision time.

The collision time τ is the time the electron "'feels"' the acceleration in each potential well (estimated before) and $\frac{1}{\tau}$ is, according to Eq.(53), the maximum angular frequency. For $\omega > \frac{1}{\tau}$ the energy irradiated per unit of frequency interval (i.e the frequency spectrum) fall rapidly to zero. This spectrum will have a cutoff at that frequency and higher frequency photons will practically not be emitted [13]. Then the spectrum will be something like a step function with the cutoff in $\frac{1}{\tau}$, as in Fig.7 (see too [14] Fig. 15.1.).

The "'height of the step"', call it $I(0)$, i.e. the intensity at zero frequency, can be obtained by re-writing Eq. (33) as

$$\delta E \delta t = \frac{4}{3}\frac{\alpha\hbar}{c^2}(\Delta v)^2 \qquad (54)$$

and being the frequencies of the photons tending to zero we can write $\delta\nu \cong \frac{1}{\delta t}$, and we have in that limit:

$$I(0) \equiv \frac{dE}{d\nu} = \frac{4}{3}\frac{\alpha\hbar}{c^2}(\Delta v)^2\,, \qquad (55)$$

or equivalently

---

[5] Strictly we call *collision time* to the time during which the electron "'feels"' the potential. That occurs in two stages, first acceleration and then deceleration, each lasting τ. In each of these lapses there is a change in the velocity Δv. This is the characteristic time that will define the frequency of cutoff of the spectrum, see below.

Valley	$\omega_c = \frac{1}{\tau}$	λ_c	$I(0)$
1	$\frac{1}{2.8\times10^{-11}s} = 3.57\times10^{10}Hz$	$8.4\,mm$	$1.63\times10^{-27}\frac{eV}{Hz}$
2	$\frac{1}{7.01\times10^{-11}s} = 1.43\times10^{10}Hz$	$2.1\,cm$	$1.03\times10^{-27}\frac{eV}{Hz}$
3	$\frac{1}{1.02\times10^{-10}s} = 9.8\times10^{9}Hz$	$3.06\,cm$	$2\times10^{-28}\frac{eV}{Hz}$
4	$\frac{1}{1.09\times10^{-10}s} = 9.17\times10^{9}Hz$	$3.27\,cm$	$1.69\times10^{-28}\frac{eV}{Hz}$

TABLE I. Characteristics of the emission spectrum (Fig. 7),i.e. cutoff frequency ω_c; minimum wavelength λ_c; and intensity $I(0)$ for each valley crossed by the electron.

$$I(0) \equiv \frac{dE}{d\nu} = \frac{4}{3}\frac{\alpha\hbar}{m^2c^2}(\nabla Q)^2\,\tau^2 \tag{56}$$

which represents the energy irradiated per unit of frequency at very low frequencies.

So for each valley there is a spectrum as in Fig.7 each one of them with a cutoff angular frequency $\omega_{ci} \equiv \frac{1}{\tau_i}$ (wavelength λ_{ci}) and "'height of the step"' $I(0)_i$ ($i = 1, 2, 3, 4$) as indicated in the table I.

The total spectrum should be composed of the superposition of step-type spectra (Fig.7), one for each valley of the quantum potential that is crossed by the electron, and each one with its cutoff frequency given above and with its corresponding "height" (Eq.56).

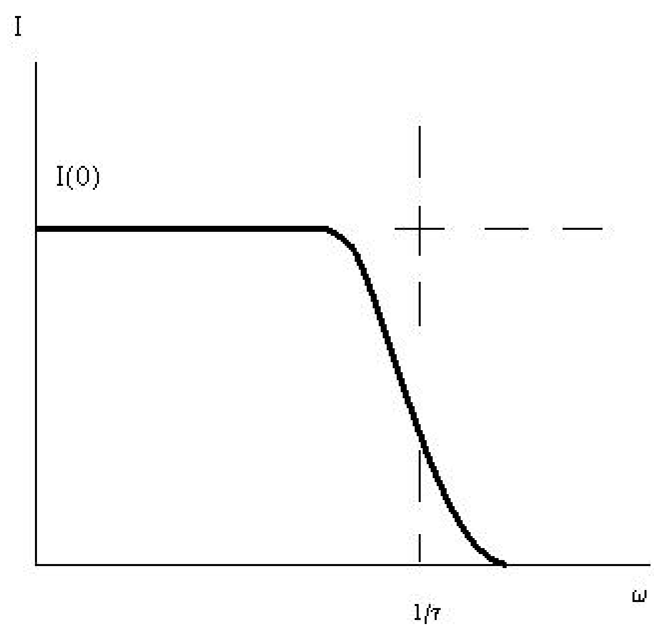

FIG. 7. Qualitative frequency spectrum for the emitted photons when the electron cross each well of the quantum potential. Here, I(0) is given by Eq.(55).

The results obtained correspond to the case in which a single electron emits radiation, it means a geometric arrangement like the Jönsson's experiment but in which the current is very weak. For example, a current as in the Tonomura's experiment in which it is used a current $I_T = 1.6\times10^{-16}A$ or $10^3 electron/s$ arriving to the screen. Under such conditions, the average distance between successive electrons is 150 Km and therefore, there is practically no chance for two electrons to be present simultaneously between the slits and the screen [3]. In such a way the values found here are compatible with an actual experiment i.e, Jönsson 's experiment with current I_T.

On the other hand, for the case of a greater current, say density current of electrons equal to $j = 30\frac{mA}{cm^2}$, exactly like the one used in the Jönsson experiment, it is possible to estimate the emission power in the following way. Using that

$$1.6022\times10^{-19}C = 1e^- \tag{57}$$

where e^- is the electron charge, we have

$j = 1.87245\times10^{17}\frac{e^-}{s.cm^2}$.

And, if we consider that the total area, S, through which that density current flows corresponds to the area of the slits with size equal to: $0.3\mu \times 50\mu$ each one [2], we have

$S = 2\times$ slit area $= 2\times0.3\times10^{-4}cm\times50\times10^{-4}cm = 3\times10^{-7}cm^2$,

and the total current I_J (where J stands for Jönsson) is

$$I_J = j.S = 1.87245\times10^{17}\frac{e^-}{s.cm^2}.3\times10^{-7}cm^2 \cong 5.6\times10^{10}\frac{e^-}{s} \tag{58}$$

On the other hand we can write (40) as

$$P_2 = 3.27\times10^{-26}W = 3.27\times10^{-26}V.\frac{C}{s}\,, \tag{59}$$

so, using (57) we can write

$$P_2 = 3.27\times10^{-26}V\times6.24\times10^{18}\frac{e^-}{s} = 2.04\times10^{-10}V\times10^3\frac{e^-}{s} \tag{60}$$

and then

$$P_2 = 2.04\times10^{-10}V\times I_T\,. \tag{61}$$

i.e the power is proportional to the current so, for the experiment with current I_J, we can write

$$P_{2J} = 2.04\times10^{-10}V\times I_J = 2.04\times10^{-10}V\times5.6\times10^{10}\frac{e^-}{s} = 1.83\times10^{-18}W = 1.83\times10^{-11}\frac{erg}{s}\,. \tag{62}$$

In the same way, from Eq.(43), it can be obtained for the 1st valley:

$$P_{1J} \cong 1.82 \times 10^{-17} W = 1.82 \times 10^{-10} \frac{erg}{s}\,, \tag{63}$$

And so on for the others wells. In this way in the case of a current as Jönsson's experiment it is obtained an emission power several orders of magnitude greater than that obtained before for a current as Tonomura's experiment and therefore, the emitted radiation will have a greater probability of being detected in an experiment like the one described.

IV. ANGULAR DISTRIBUTION WITH A DEFINITE STATE OF POLARIZATION

We can say something in relation to the angular distribution of the emitted radiation and its polarization. The key in this experiment is that the acceleration imparted by the quantum potential to the electron has only a component in the "y" direction. Therefore, making use of the results presented in [13] it is possible to see that for the angular spectral distribution, I, will survive only the contribution due to this polarization direction, i.e $I_{\perp}$, see Eq. (15.10) of that reference (remember that they are also valid from the quantum point of view). We will leave for a next article the details of this issue.

V. CONCLUSION

It has been demonstrated from the usual quantum mechanics (interpretation of Copenhagen), that in the two slits experiment of interference with electrons they do not emit radiation in their way from the slits to the screen, i.e. the probability amplitude for that process is zero. On the other hand, using the quantum mechanics in the view of Bohm-de Broglie, we have shown that these electrons must emit radiation. The reason for this emission is that the quantum potential accelerates the electrons. For realistic experimental parameters compatible with experiments already carried out, we have shown that the emission spectrum is a superposition of step functions, each of them characterized by a cutoff frequency and a certain intensity. They are radio waves with wavelengths that go approximately from cm onwards, of very low power, but probably detectable. If this were the case, a measure that could detect this spectrum of emitted radio waves would constitute a strong experimental evidence of the existence of Bohmian trajectories. Note that Chen's prediction [8] indicates electromagnetic waves in the visible range (using slits of another thickness) although in truth, as we observed earlier, the same author has completely refuted his own prediction [9].

Zero emission is the first and main response that the Copenhagen vision can give and, as we saw, it is different from the prediction that results from the BdB vision. However, we think that Copenhagen's prediction could be another in the light of the following argument. The electron, when coming out of the slits, is not in principle subjected to any potential as to do not cancel the equation (30). However, from the point of view of quantum field theory, there is a zero point energy, that is, the energy of the vacuum. The possibility that we consider is that the electron is subjected to the fluctuations of energy of the vacuum, which would be variable with the position in a way that produces for the commutator, Eq. (30), a non-zero value of the operator $\ddot{\boldsymbol{r}}$, in the same way as the quantum potential in the BdB view. If this were possible, we could speak of some sort of equivalence of the quantum potential with the fluctuations of the energy of the vacuum, and the predictions of both visions could be the same, although this must be investigated.

In this work, given the complexity of the quantum potential, we used graphic methods, somewhat handcrafted, which we believe are justified given the novelty of the result obtained. Nevertheless, we think it is convenient to improve these results using numerical developments for the quantum potential, task which will be the subject of a forthcoming investigation.

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VII. APPENDIX

A. Computation of equation (25)

$$\dot{\boldsymbol{r}} = \frac{i}{\hbar}[H, \boldsymbol{r}] = \frac{i}{\hbar}[H_e + H_{rad}, \boldsymbol{r}] = \frac{i}{\hbar}[H_e, \boldsymbol{r}] = \frac{i}{\hbar}(H_e \boldsymbol{r} - \boldsymbol{r} H_e) \tag{64}$$

then

$$\begin{aligned} \langle p_b | \boldsymbol{p} | p_a \rangle &= m \langle p_b | \dot{\boldsymbol{r}} | p_a \rangle = \\ & m \langle p_b | H_e \boldsymbol{r} - \boldsymbol{r} H_e | p_a \rangle \frac{i}{\hbar} = \\ & m (E_b \langle p_b | \boldsymbol{r} | p_a \rangle - \langle p_b | \boldsymbol{r} | p_a \rangle E_a) \frac{i}{\hbar} = \\ & \frac{im}{\hbar} (E_b - E_a) \langle p_b | \boldsymbol{r} | p_a \rangle \\ &= im\omega_{ba} \langle p_b | \boldsymbol{r} | p_a \rangle \end{aligned} \tag{65}$$

it means that

$$\langle p_b | \boldsymbol{p} | p_a \rangle = im\omega_{ba} \langle p_b | \boldsymbol{r} | p_a \rangle \;\; i.e. \tag{66}$$

$$\langle p_b | \dot{\boldsymbol{r}} | p_a \rangle = i\omega_{ba} \langle p_b | \boldsymbol{r} | p_a \rangle\,. \tag{67}$$

Now in the same way we can compute $\langle p_b | \ddot{\boldsymbol{r}} | p_a \rangle$ and obtain:

$$\langle p_b | \ddot{\boldsymbol{r}} | p_a \rangle = i\omega_{ba} \langle p_b | \dot{\boldsymbol{r}} | p_a \rangle \quad (68)$$

and using (67) we have

$$\langle p_b | \ddot{\boldsymbol{r}} | p_a \rangle = -\omega_{ba}^2 \langle p_b | \boldsymbol{r} | p_a \rangle \quad (69)$$

that together (66) allow us write (25).

B. Showing the plausibility of (33)

It is possible to "'re-obtain' equation Eq. (29) by following the elementary considerations given by Thirring in [15] page 7 : an electron which follows an accelerated movement must emit radiation according to classical electrodynamics. But from the quantum theoretical point of view we can only say that there exist a certain probability for the accelerated electron emit a photon [6].

If the electron changes its velocity $\boldsymbol{v}$ in $\Delta\boldsymbol{v}$ during the time interval Δt, the photon emission probability w is given in essence by the Larmor formula by

$$w \sim \alpha(\Delta\boldsymbol{v})^2 \quad (70)$$

where α is fine structure constant.

The energy emitted by this electron is, on the average, equal to the product of probability by the energy of the emitted photon[7]

$$\Delta E \sim \alpha(\Delta\boldsymbol{v})^2 \frac{\hbar}{\Delta t} \quad (71)$$

where the frequency of the photon is of the order $\frac{1}{\Delta t}$.

Then, for the emitted power we have

$$\frac{\Delta E}{\Delta t} \sim \alpha\hbar \left(\frac{\Delta\boldsymbol{v}}{\Delta t}\right)^2 \quad (72)$$

or in infinitesimal form

$$\frac{dE}{dt} \sim \alpha\hbar \left(\frac{d\boldsymbol{v}}{dt}\right)^2 \quad (73)$$

which are in essence Eq.(32) and Eq. (33) respectively, except for constants.

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[6] We consider the single emission of a photon because it is much more likely than multiple emissions, see [15]

[7] Usually

$$\langle E \rangle \equiv \int E\, |\psi(E)|^2\, dE \Rightarrow d\langle E \rangle = |\psi(E)|^2\, dE.E = probability \times energy$$